From

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To

Head of Department
Department of Mathematics
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Respected Mam,

As I went to write SSC Exam (11.09.2024) I was unable to attend the Graph theory. internal exam. So I kindly request you to conduct retest on another day. (15.09.2024).

Thanking You

Yours obediently,
Akshaya Mol. S

S. Akshya
No: 2
I MSc Maths

Graph thoory and Applications

11/20

Part-D

6. a) Proof:

Suppose G is a bipartite graph with bipartition (X, Y).

Let $C = (v_1 e_1, v_2 e_2, v_3 e_3, \ldots \ldots v_k e_k v_1 e_1)$ be the cycle of G.

without loss of generality, we can suppose that $v_1 \in X$, v_2 is adjacent to v_1, $v_2 \in Y$.

Let $v_i \in X$ according to the odd and even cycle. $1 \leq i \leq k$,

Now, $v_1 v_k \in X$ no edges of G.

$v_1 \in X$ and $v_k \in Y$ according to the odd and even cycle.

Conversely we assume that G contains no odd cycle.

∴ G is connected.

~~Let X defined X~~

Let v be the vertices of G.

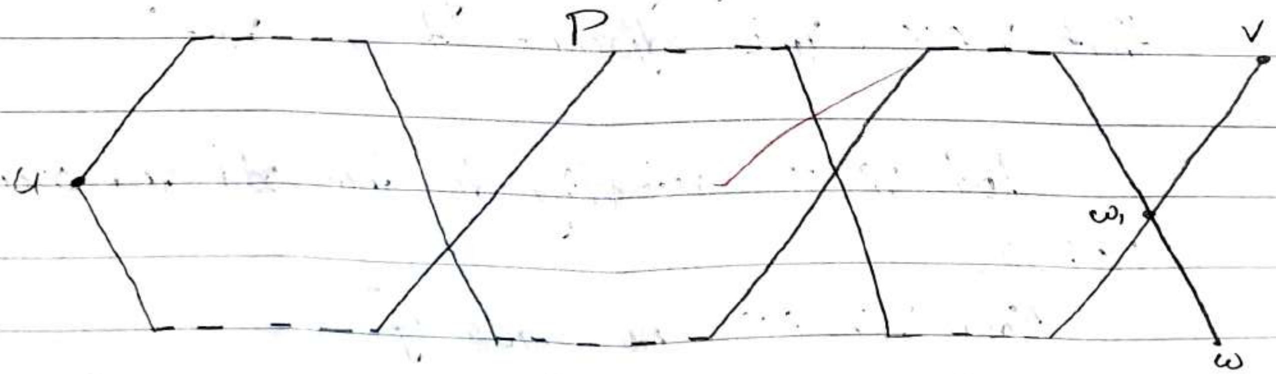
Defind $X = \{v \in V \mid d(u,v)$ is even

~~Also~~, $Y = \{v \in V \mid d(u,v)$ is odd.

We will prove that G is the ~~pa~~ bipatite graph of (X,Y)

ie, to prove, no two vertices of X and no two vertices Y are adjacent in G.

Let uw be the path, $d(u,w)$, $d(v,w)$ ~~be~~



Now uw_1 be the path of the P and Q have the same length. p and q

w_1-v be the section of shortest length q and ww_1 be the section of shortest length p.

u,w be the section of P and q have same length.
w, v be the section of P have the shortest length p and w, w be the section of Q have the shortest length q.

w, v and w, u the be the sam section of same length with no common vertex.

If $e = vw$, there is no edges followed by X and Y.
$\therefore \phi'$
Which is contradiction

No two vertices of X is adjacent in G.
Simillarly,
No two vertices of Y is adjacent in G.

$\therefore$ G is a bipartite.

Suppose G is not connected.

Let $G = G_1, G_2, \dots G_w$ be the partition of G.

Let X_i, Y_i be the bipartite of G_i

Then, $X_i = \bigcup_{i=1}^{\omega}$ and $Y_i = \bigcup_{i=1}^{\omega}$ be the bipatitite,

$\therefore$ G is a bipartitiae.

$\therefore$ G contains no odd cycles.

$\therefore$ G is a bipartite.

Part-B

5-a)

4. If G is simple and $\delta \geq \frac{n-1}{2}$, then G is connected

Proof:

Suppose G is connected then G_1 and G_2 be the two components of G.

Let v be the vertices of G_1

$$\delta \geq \frac{n-1}{2}$$
$$d(v) = \frac{n-1}{2}$$

G_1 contains $d(v)+1 = \frac{n-1}{2} + 1$

$$d(v) = \frac{n+1}{2}$$

G_1 contains $d(v)+1 = \frac{n-1}{2}+1$

$$d(v) \cancel{+1} = \frac{n+1}{2}$$

$\therefore$ G_1 and G_1 are $(n+1)$ $(n+1)$ vertices.

$\therefore$ ~~G is connected~~

Which is contradiction,

$\therefore$ G is connected.

Part C

5.b) Proof:

Let ~~in a~~ Connected Graph G with atleast three vertices, any two longest path have a vertex in Common.

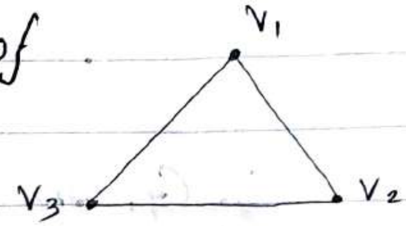
Now to prove G is connected,

Conversely suppose that G is not connected,

Let $V = v_1, v_2, \dots, v_n$ be the vertices of G.

Let v_1, v_2, v_3 be the least three vertices

v_1, v_2, v_3 be the three vertices of three vertices Graph G



v_1 and v_2 are two vertices ~~and~~ with ~~two~~ edges
v_1 and v_3 are two vertices with edges

Now, v_1 be the common vertex of the v_2 and v_3

$\therefore$ The longest path ~~of~~ have a vertex in common

Which is contradiction.
$\therefore G$ is connected.

Q. Soln:
Q.